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Abstract. There are many possible gravitational applications of an effective approach to Quantum Field Theory (QFT) in curved space. We present a brief review of effective approach and discuss its impact for such relevant issues as the cosmological constant (CC) problem and inflation driven by vacuum quantum effects. Furthermore it is shown how one can impose significant theoretical constraints on a non-metric gravity using only theoretical effective field theory framework.

1 Introduction

The effective approach to QFT implies the low-energy phenomena being described independently on the (sometimes unknown) fundamental physics. A famous example is the low-energy QCD, where the Chiral Perturbation Theory helps to achieve results fitting both lattice simulations and experiment, in a situation when usual perturbative methods are not applicable. There are many good reviews on the effective approach, see e.g. [1] and references therein.

Sometimes, the effective approach enables one to reduce the requirements to a theory, e.g. extract relevant low-energy information even from the non-renormalizable theories, where the high-energy UV limit is problematic. It might happen that this kind of considerations is applicable even to quantum gravity, e.g. one can extract from this theory the long-distance quantum correction to the Newton potential [2]. The theoretical relevance of this result can not be questioned (even in spite of the extreme smallness of the effect), because it helps in better understanding of the consistency conditions for the effective approach, e.g. the importance of the full set of Feynman diagrams [3, 4]. However, despite the theoretical investigation of Quantum Gravity is really fascinating, we can not be sure that they have something to do with Nature. It might happen that the gravitational field should not be quantized at all, representing just a classical background for the quantized matter fields and particles.

In the present review we shall concentrate on the effective approach to QFT in curved space. This is perhaps the most natural way of investigating quantum and gravitational phenomena together [5, 6]. The enormous success of QFT in describing the theories like QED or Standard Model prove the validity of QFT methods. On the other hand, since there are many indications that our space-time is curved, the QFT in a curved space is, let us say, a correct theory applied in a correct place. Our main point is that, using the effective approach and extracting relevant information at different energy scales, one has a chance to learn a natural description for many interesting phenomena, such as inflation or scale (or time) dependence of the vacuum energy.

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Moreover, while making quantitative predictions is sometimes difficult yet, one can exclude some definite options for the space-time using effective approach.

The important aspect of effective approach is the notion of decoupling. At classical level decoupling means that a heavy field doesn't propagate at low energies. This can be easily seen observing the propagator of a massive particle at low energy

$$\frac{1}{k^2 + M^2} \approx \frac{1}{M^2} + \mathcal{O}\left(\frac{k^2}{M^4}\right), \quad k^2 \ll M^2. \quad (1)$$

For example, if we take a gravity theory with higher derivatives, the typical value of M would be M_P . Already for $|k| = 1 \text{ TeV}$ the ratio $k^2/M^2 \propto 10^{-32}$. It is clear that only massless mode of gravity is relevant even for the early Universe or other potentially observable phenomena. The important exception is inflation, which will be discussed below.

The decoupling theorem explains how the suppression of the effects of heavy particles occurs at quantum level. Consider the QED example in flat space. The 1-loop vacuum polarization is

$$-\frac{e^2 \theta_{\mu\nu}}{2\pi^2} \int_0^1 dx x(1-x) \ln \frac{m_e^2 + p^2 x(1-x)}{4\pi\mu^2}, \quad (2)$$

where $\theta_{\mu\nu} = (p_\mu p_\nu - p^2 g_{\mu\nu})$ and μ is the parameter of dimensional regularization. The minimal subtraction scheme of renormalization results in the μ -dependence for the relevant observables. For instance, the β -function $\beta^{\overline{\text{MS}}}$ results after acting $\frac{\epsilon}{2}\mu \frac{d}{d\mu}$ on the formfactor of $\theta_{\mu\nu}$

$$\beta_e^{\overline{\text{MS}}} = \frac{e^3}{12\pi^2}. \quad (3)$$

This β -function can not tell us about the decoupling and possesses an artificial universality, because the $\overline{\text{MS}}$ scheme is efficient only in the UV limit. In order to define physical β -function, let us apply the mass-dependent renormalization scheme. Subtracting the divergence at $p^2 = M^2$ and taking derivative $\frac{\epsilon}{2}M \frac{d}{dM}$ we arrive at

$$\beta_e = \frac{e^3}{2\pi^2} \int_0^1 dx x(1-x) \frac{M^2 x(1-x)}{m_e^2 + M^2 x(1-x)}.$$

The UV limit ($M \gg m_e$) gives $\beta_e = \beta_e^{\overline{\text{MS}}}$ and in the IR limit ($M \ll m_e$) we meet a quadratic decoupling (Appelquist & Carazzone theorem [7])

$$\beta_e = \frac{e^3}{60\pi^2} \cdot \frac{M^2}{m_e^2} + \mathcal{O}\left(\frac{M^4}{m^4}\right). \quad (4)$$

Compared to $\beta_e^{UV} = \beta_e^{\overline{\text{MS}}}$, in the IR there is a suppression of the quantum contribution.

2 Decoupling and cosmological constant

Quantitative investigation of decoupling for quantum matter on curved background started recently [8]. Why this is interesting and important? The most explicit example is the decoupling of the quantum contributions to the CC (vacuum energy) [9]. Let us assume that the Appelquist &

Carazzone-like quadratic decoupling holds for a CC and consider the consequences for the present-day Universe. One has to associate the scale $\mu \equiv H$ with the Hubble parameter [10]. Let us notice that this identification of the scale has serious advantages over other possible choices [11, 12].

Remember that, in the $\overline{\text{MS}}$ scheme, $\beta_\Lambda \sim m^4$, m being mass of a quantum matter field [6]. Then the quadratically suppressed expression is [9]

$$H \frac{d\Lambda}{dH} = \beta_\Lambda = \sum_i c_i \frac{H^2}{m_i^2} \times m_i^4 = \frac{\sigma}{(4\pi)^2} M^2 H^2, \quad (5)$$

where M is an unknown mass parameter and $\sigma = \pm 1$. The expression σM^2 is the algebraic sum of the contributions of all virtual fields, the ones of the heaviest particles being the most relevant. The sign σ depends on whether fermions or bosons dominate in the particle spectrum.

Let us assume, for a moment, that $M^2 = M_P^2$. Then we find

$$|\beta_\Lambda| \sim 10^{-47} \text{ GeV}^4, \quad (6)$$

that is close to the existing supernovae and CMB data for the vacuum energy density. Therefore, the renormalization group may, in principle, explain the variation of the vacuum energy with the change of the scale (which, in presence of matter, means also variation of time) without introducing special entities like quintessence.

It is clear that the renormalization group can not solve the famous CC problem [15], neither the coincidence problem. The fine-tuning of the vacuum CC is performed at the instant when we define the initial point Λ_0 of the renormalization group flow for Λ [10]. However, the running of the CC may perform in the range comparable to the observed CC, making the coincidence problem less severe.

The example of a cosmological model with running CC has been developed in [9]. Along with the renormalization group equation (5), there is also Friedmann equation (for simplicity we consider $k = 0$ here)

$$H^2 = \frac{8\pi G}{3} (\rho + \Lambda), \quad (7)$$

and the conservation law. The last can be chosen in many possible ways. The choice

$$\frac{d\Lambda}{dt} + \frac{d\rho}{dt} + 3H(\rho + P) = 0 \quad (8)$$

means we admit the energy exchange between the vacuum and matter sectors (see also [13], where similar cosmological model with $\Lambda_0 = 0$ have been developed earlier). The solution of the equations (5), (7), (8) is analytical for any equation of state $P = \alpha\rho$. In terms of the red-shift parameter $z = a_0/a - 1$ this solution has the form

$$\rho(z; \nu) = \rho_0 (1+z)^{3(1-\nu)(\alpha+1)}, \quad \Lambda(z; \nu) = \Lambda_0 + \frac{\nu}{1-\nu} [\rho(z; \nu) - \rho_0], \quad (9)$$

where ρ_0, Λ_0 are the present day values, and $\nu = \sigma M^2/12\pi M_P^2$. When $\nu \rightarrow 0$ we recover the standard result for $\Lambda = \text{const.}$

The value of $|\nu|$ has to satisfy the constraint $|\nu| \ll 1$, for otherwise there is a problem with the nucleosynthesis [9]. The “canonical” value $M^2 = M_P^2$ gives $|\nu| = \nu_0 \simeq 2.6 \times 10^{-2}$, compatible with this constraint. The next question is whether the permitted values $\nu \ll 1$ may lead to observable consequences. The answer is positive. For example, the relative deviation

$$\delta_\Lambda(z; \nu) = \frac{\Lambda(z; \nu) - \Lambda_0}{\Lambda_0}$$

can be evaluated in the linear in ν approximation with respect to the red shift z_0 corresponding to the existing supernovae data [16]

$$\delta_\Lambda(z; \nu) = \frac{\nu \Omega_M^0}{\Omega_\Lambda^0} [(1+z)^3 - (1+z_0)^3]. \quad (10)$$

Taking $z_0 \simeq 0.5$ with $\Omega_M^0 = 0.3$ and $\Omega_\Lambda^0 = 0.7$, and $\nu = \nu_0$, we find $\delta_\Lambda(z = 1.5; \nu_0) \approx 14\%$, that is a potentially measurable effect. It is remarkable that the cubic relation (10) holds also for another choice of the conservation law [14].

In fact, the QFT methods can teach us more lessons about the form and origin of quantum corrections to the CC [17]. Let us remind that behind the renormalization group there is a well-defined object called Effective Action of vacuum (EA)

$$e^{i\Gamma[g_{\mu\nu}]} = \int \mathcal{D}\Phi e^{iS[\Phi, g_{\mu\nu}]}. \quad (11)$$

Typically, $\Gamma[g_{\mu\nu}]$ is a complicated non-local functional of the background metric. The renormalization group reflects the scaling dependence of EA. When considering the low-energy cosmological applications, we safely can perform the expansion in the Hubble parameter H similar to the one of (1). Let us remark that the masses of all particles are many orders of magnitude smaller than the present day value $H_0 \approx 10^{-42} \text{ GeV}$. E.g. the neutrino masses are about 10^{-12} GeV while the QCD scale $\Lambda_{QCD} \approx 10^{-2} \text{ GeV}$. Hence the H expansion is just a reliable way to parametrize all non-localities including the ones related to the non-perturbative QFT effects (non-perturbative in couplings but not in H) [9].

At present, our ability of deriving an explicit form of EA for the massive fields is very restricted. But we are certain it is a covariant functional [6], and even this knowledge is sufficient for obtaining an essential piece of information. Due to covariance, $\Gamma[g_{\mu\nu}]$ must be even in metric derivatives. In the cosmological setting this means that the linear in H quantum corrections to the CC are completely ruled out. Of course, the last statement is valid only for the case when metric is the unique background field. If there is an extra light (with the mass comparable to H_0) field φ , such as quintessence, then the EA is $\Gamma[g_{\mu\nu}, \varphi]$ and any kind of a functional dependence between vacuum energy and H may take place. However, without quintessence no QFT can produce $\mathcal{O}(H)$ quantum corrections. Let us stress that this restriction is valid independently on the nature of quantum corrections (local, non-local, perturbative or non-perturbative ones).

Without quintessence the quantum corrections start from H^2 and therefore, e.g. the QCD vacuum effects play no role in cosmology. The relevant quantum effects on the vacuum energy may come only from the Planck scale physics or, at least, from the GUT-scale physics.

3 Higher derivative sector

In order to derive decoupling theorem for gravity we have considered massive fields on the classical metric background [8]. Unfortunately there is no completely covariant technique compatible with the mass-dependent renormalization schemes. Hence we can perform calculations only for the linearized gravity on the flat background $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$. The corrections to the graviton propagator come from the Feynman diagrams at the Figure 1.

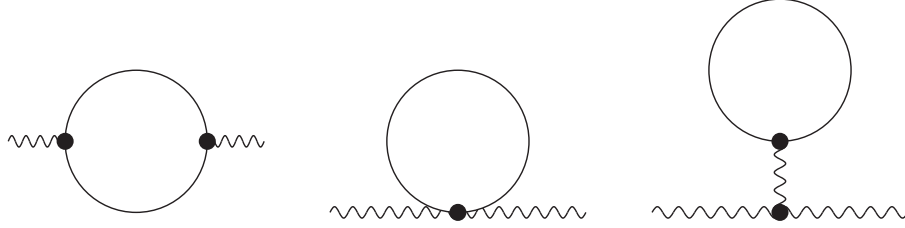


Figure 1. Wavy lines mean $h_{\mu\nu}$ and solid lines quantum matter field.

The polarization operator must be compared to the tensor structure of the Lagrangian

$$L = -\Lambda - \frac{1}{16\pi G} R + a_1 C^2 + a_2 E + a_3 \square R + a_4 R^2, \quad (12)$$

where $C^2 = C_{\mu\nu\alpha\beta}^2$ is the square of the Weyl tensor and E is the integrand of the Gauss-Bonnet topological invariant $E = R_{\mu\nu\alpha\beta}^2 - 4R_{\mu\nu}^2 + R^2$.

An alternative equivalent way of calculation is using the heat kernel solution in the second order in curvature approximation [18, 19].

For the formfactors we find, e.g. for the real scalar field

$$k_\Lambda = \frac{3m^4}{8(4\pi)^2}, \quad k_R = \frac{m^2}{2(4\pi)^2} \left(\xi - \frac{1}{6} \right), \quad k_1(a) = \frac{8A}{15a^4} + \frac{2}{45a^2} + \frac{1}{150}, \quad (13)$$

where

$$A = 1 + \frac{1}{a} \ln \left| \frac{2-a}{2+a} \right|, \quad a^2 = \frac{4\square}{4m^2 - \square}.$$

Obviously, constant formfactors mean zero β -functions for the CC and G . At the same time for the coefficient of the Weyl term we find

$$\beta_1 = -\frac{1}{(4\pi)^2} \left(\frac{1}{18a^2} - \frac{1}{180} - \frac{a^2 - 4}{6a^4} A \right). \quad (14)$$

Then

$$\beta_1^{UV} = -\frac{1}{(4\pi)^2} \frac{1}{120} + \mathcal{O}\left(\frac{m^2}{p^2}\right), \quad \text{and} \quad \beta_1^{IR} = -\frac{1}{1680(4\pi)^2} \cdot \frac{p^2}{m^2} + \mathcal{O}\left(\frac{p^4}{m^4}\right). \quad (15)$$

The last formula is nothing but the Appelquist & Carazzone theorem for gravity. Our calculations [8] have shown it holds in all higher derivative sector, including the theories with the Spontaneous Symmetry Breaking [20].

An expansion $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$ works well for higher derivative terms, but not for Λ and G . Why did we obtain $\beta_\Lambda = \beta_{1/G} \equiv 0$? In fact, running means the presence of a $f(\Box) = \ln(\Box/\mu^2)$ -like formfactor. In QED, in the UV limit we meet the term

$$-\frac{e^2}{4}F_{\mu\nu}F^{\mu\nu} + \frac{e^4}{3(4\pi)^2}F_{\mu\nu}\ln\left(-\frac{\Box}{\mu^2}\right)F^{\mu\nu}.$$

Similarly in gravity it is possible to insert

$$C_{\mu\nu\alpha\beta}f(\Box)C^{\mu\nu\alpha\beta} \quad \text{or} \quad Rf(\Box)R$$

in the higher derivative sector. However, no insertion is possible for Λ and $1/G$, since $\Box\Lambda = 0$ and $\Box R$ is a total derivative.

Does it mean that β_Λ and $\beta_{1/G}$ really equal zero? From my point of view the answer is negative, for otherwise we meet a divergence between the mass-dependent renormalization scheme and \overline{MS} -scheme in the UV where they are supposed to be the same. Perhaps calculations on a flat background are not appropriate for deriving the renormalization group equations for Λ and $1/G$. This hypothesis is quite natural, especially because flat space is not a classical solution in the presence of the CC.

4 Anomaly-induced inflation

At present, we are not able to derive or disprove the relation (5) and hence the cosmological model with variable CC described in section 2 is essentially phenomenological. What we can calculate is the decoupling in the higher derivative sector. In fact, this is also very interesting, creating a solid basis for the anomaly-induced inflation (modified Starobinsky model) [21, 22].

The most important result is the β_3 -function in the theory with broken supersymmetry. Due to the decoupling of the heavy sparticles this β -function smoothly changes its sign from negative in the UV to positive in IR [8]. The β_3 -function is nothing else but the coefficient c in the expression for the conformal anomaly

$$T = \langle T_\mu^\mu \rangle = -(wC^2 + bE + c\Box R), \quad (16)$$

where w, b, c depend on the number of fields of different spins $N_0, N_{1/2}, N_1$ in the underlying GUT. The signs of other two coefficients $w > 0$ and $b < 0$ are universal such that only the sign of c alternates.

Taking (16) into account we arrive at the equation for the conformal factor of the metric

$$\frac{\ddot{a}}{a} + \frac{3\dot{a}\ddot{a}}{a^2} + \frac{\ddot{a}^2}{a^2} - \left(5 + \frac{4b}{c}\right)\frac{\ddot{a}\dot{a}^2}{a^3} - \frac{M_P^2}{8\pi c}\left(\frac{\ddot{a}}{a} + \frac{\ddot{a}^2}{a^2} - \frac{2\Lambda}{3}\right) = 0, \quad (17)$$

The important particular solutions are

$$a(t) = a_0 e^{Ht}, \quad H^2 = -\frac{M_P^2}{32\pi b}\left(1 \pm \left[1 + \frac{64\pi b \Lambda}{3M_P^2}\right]^{1/2}\right). \quad (18)$$

For $0 < \Lambda \ll M_P^2$ there are two solutions:

$$H_f \approx \sqrt{\Lambda/3}, \quad \text{and} \quad H_i \approx \sqrt{-\frac{M_P^2}{16\pi b} - \frac{\Lambda}{3}} \approx \frac{M_P}{\sqrt{-16\pi b}}, \quad (19)$$

the last one corresponds to the Starobinsky inflation [23]. The inflationary solution is stable under perturbations of $\sigma(t) = \ln a(t)$ in case the condition

$$c \sim \frac{N_0}{18} + \frac{N_{1/2}}{3} - N_1 > 0 \quad (20)$$

is satisfied [23, 21]. If the gauge model includes many fermions and scalars for a given number of vectors, its vacuum quantum effects lead to the stable inflation, otherwise the inflation is unstable and there is a chance of a graceful exit to the FRW-like evolution. The original Starobinsky model [23] is based on the unstable case and involves heavy fine-tunings for the initial data. Our purpose is to avoid fine-tunings at all, fortunately the effective approach shows this is possible.

In order to understand how one can consider both stable and unstable inflations in the very same model, let us assume that at the UV ($H \gg M_F$) there is supersymmetry, e.g. MSSM with a particle content

$$N_1 = 12, \quad N_{1/2} = 32, \quad N_0 = 104.$$

This provides stable inflation, because $c > 0$. Similar situation holds for any realistic SUSY model. The advantage of stable inflation is that it is independent on the initial data. But why should inflation end?

Already at the MSM ($N_{1,1/2,0} = 12, 24, 4$) scale $H \sim 10^2 \text{ GeV}$ the inflation is unstable, $c < 0$. One can suggest the following physical interpretation of this sign difference. We know that all sparticles are heavy, hence they decouple, when H becomes smaller than their masses. According to [8], the transition from $c > 0$ to $c < 0$ is smooth, giving a hope for a graceful exit.

The next question is why the magnitude of the Hubble parameter is becoming smaller in the course of inflation. An explicit calculation of the contribution of a particle of mass m in the regime $H \gg m$ has been performed in [22]. Extrapolating the result until $H \approx m$ we arrive at the curve $\sigma(t)$ shown at the Figure 2.

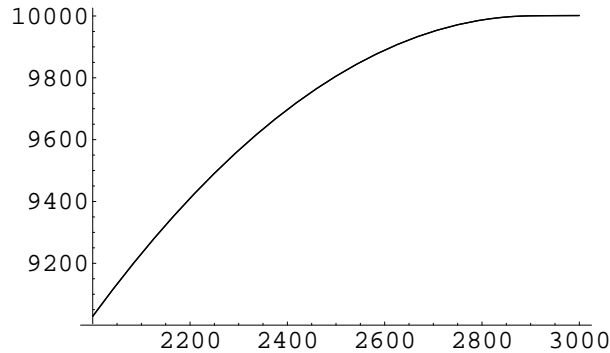


Figure 2. The plot of $\sigma(t) = \ln a(t)$. An approximate analytic expression is $\sigma = H_i t - H_i^2 \tilde{f} t^2 / 4$. The value of \tilde{f} depends on the mass spectrum of the theory. For MSSM the total amount of e -folds is as large as 10^{32} , but only 65 last ones, where $H \propto M_*$ (SUSY decoupling scale) are relevant.

An additional advantage of the anomaly-induced inflation becomes clear when investigating the gravitational waves in this model. It turns out that, during the last 65 e -folds of inflation the production of these waves is restricted and they almost do not amplify [24, 21] (see also [25]). Again, this nice feature follows without any artificial restrictions or fine-tuning.

As we can see, the anomaly-induced model represents a strong candidate to the role of natural inflation scenario, because it does not require fine-tuning for initial data and does not need a special entity such as inflaton. However, small definite information is available about the most important intermediate stage of inflation. In order to obtain this information one needs further development of QFT in curved space-time, mainly investigating the vacuum contributions of massive fields on a dynamical metric background.

5 Restrictions on Space-Time Geometry from effective QFT

Consider how one can impose the restrictions on a Space-Time Geometry using effective approach to QFT. In fact, we can impose very rigid constraints on the propagation of a space-time torsion in the effective framework [26, 27, 28].

The theories of gravity with torsion attracted significant attention for a long time (see e.g. [29, 28] and references therein). Torsion $T^\alpha_{\beta\gamma}$ is independent (on metric) characteristic of a space-time manifold which is defined by the relation

$$\Gamma^\alpha_{\beta\gamma} - \Gamma^\alpha_{\gamma\beta} = T^\alpha_{\beta\gamma}.$$

It is useful dividing torsion into irreducible components

$$T_{\alpha\beta\mu} = \frac{1}{3} (T_\beta g_{\alpha\mu} - T_\mu g_{\alpha\beta}) - \frac{1}{6} \varepsilon_{\alpha\beta\mu\nu} S^\nu + q_{\alpha\beta\mu}. \quad (21)$$

Interaction to the Dirac fermion has the form (we assume, for simplicity, the flat metric)

$$\mathcal{L} = i\bar{\psi}\gamma^\mu(\partial_\mu + i\eta_1\gamma^5 S_\mu + i\eta_2 T_\mu)\psi + m\bar{\psi}\psi, \quad (22)$$

where η_1, η_2 are nonminimal parameters. The minimal case corresponds to $\eta_1 = 1/8, \eta_2 = 0$, so the presence of torsion means, at least, that any fermion is coupled to an axial vector S_μ

$$S = i \int d^4x \bar{\psi} \gamma^\mu (\partial_\mu + i\eta_1 \gamma^5 S_\mu - im) \psi. \quad (23)$$

Let us notice that the interaction with torsion is nothing else but the known CPT & Lorentz violating term [30]. The question is whether we can construct a QFT for ψ and S_μ which would be unitary and also renormalizable as an effective QFT. Let us consider this QFT in two steps. First, quantizing ψ we meet the two types of divergences [6, 26]:

$$S_{\mu\nu} S^{\mu\nu} \quad \text{and} \quad m^2 S_\mu S^\mu, \quad \text{where} \quad S_{\mu\nu} = \partial_\mu S_\nu - \partial_\nu S_\mu. \quad (24)$$

It is well-known that unitarity forbids simultaneous S_μ^\perp and S_μ^\parallel propagation. Therefore, the unique possibility for a dynamical torsion is [26]

$$S_{tor} = \int d^4x \left\{ -\frac{1}{4} S_{\mu\nu}^2 + M^2 S_\mu^2 \right\}. \quad (25)$$

As a second step we have to investigate whether the effective quantum theory of fermion coupled to dynamical torsion is consistent. An extremely involved calculations yield [27] a longitudinal $(\partial_\mu S^\mu)^2$ -type divergence at the two-loop level. This means there is a severe conflict between unitarity and renormalizability in the low-energy corner of the theory.

One possible solution is to assume $\eta^4 m^2 \ll M^4$. This means that either $M \gg m$ for all fermions, or that η is extremely small. In both cases there is no chance to observe propagating torsion. The lower bound from LEP is $M/\eta \leq 3 \text{ TeV}$ [26], that is far beyond the necessary condition presented above. Finally, torsion can be a composite field (e.g. it can be a vacuum condensate) but it can not be an independent and experimentally observable propagating field.

6 Conclusions

The effective approach to QFT in curved space-time may tell us a lot about gravitational physics, especially in cosmology. Perhaps the most interesting problem is evaluation of vacuum effective action for massive quantum fields. Working in this direction one may prove or disprove the possibility of a time-dependent cosmological constant. The same calculation is vital for further theoretical development of the anomaly-induced inflation model.

Within the existing formalism of QFT in curved space and known calculational techniques we can learn something relevant about the possible form of quantum corrections, e.g. exclude the $\mathcal{O}(H)$ -type corrections to the CC. If this kind of dependence will be someday detected, it will be direct indication to the existence of a qualitatively new field such as quintessence.

Surprisingly, one can exclude some relevant options (such as propagating torsion) for the space-time geometry using QFT methods and effective approach.

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